DO NOW

Recall volume formulas for rectangular prisms, cubes, cylinders, cones, spheres...

3.7 Related Rates - Day 2

Volume formulas:

rectangular prism - $V = \ell w h$

cube - $V = e^3$

cylinder - $V = \pi r^2 h$

cone - $V = \frac{1}{3}\pi r^2 h$

sphere - $V = \frac{4}{3}\pi r^3$

Example:

3. A spherical balloon is being filled with a gas in such a way that when the radius is 2 ft, the radius is increasing at the rate of $\frac{1}{6}$ ft/min. How fast is the volume changing at this time?

Find $\frac{dV}{dt}$ when $\frac{dr}{dt} = \frac{1}{6}$ ft/min, $r = 2$ ft

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(2)^2\left(\frac{1}{6}\right)$

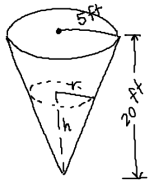
$\frac{dV}{dt} = 4\pi(4)\left(\frac{1}{6}\right)$

$\frac{dV}{dt} = \frac{16\pi}{6} = \frac{8\pi}{3}$

$\frac{8\pi}{3}$ ft^3/min

4. A tank filled with water is shaped as an inverted cone, 20 ft high with a circular base (top) whose radius is 5 ft. Water is running out of the bottom of the tank at a constant rate of 2 ft^3/min. How fast is the water level falling when the water is 8 ft deep?

Find $\frac{dh}{dt}$ when $\frac{dV}{dt} = -2$ ft^3/min, $h = 8$ ft



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2(h)$

$V = \frac{\pi}{48}h^3$

$\frac{dV}{dt} = \frac{\pi}{48}(3h^2)\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{16}h^2\frac{dh}{dt}$

$-2 = \frac{\pi}{16}(8)^2\frac{dh}{dt}$

$-2 = \frac{64\pi}{16}\cdot\frac{dh}{dt}$

$-2 = 4\pi\frac{dh}{dt}$

$\frac{-2}{4\pi} = \frac{dh}{dt}$

$-\frac{1}{2\pi} = \frac{dh}{dt}$

$\frac{1}{2\pi}$ ft/min

* You don't have r or $\frac{dr}{dt}$ ...
→ similar △'s

$\frac{r}{h} = \frac{5}{20}$

$20r = 5h$

$r = \frac{5h}{20}$

$r = \frac{h}{4}$

** replace r **

HOMEWORK

Worksheet: Homework 3.7; 8 - 18